

(i) If A is a square matrix of order 3 such that $|A| = 3$, then the value of $|\text{adj}(A)|$ is:

- (a) 3
- (b) 9
- (c) 27
- (d) 81

(ii) The degree of the differential equation $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2} = \frac{d^2y}{dx^2}$ is:

- (a) 2
- (b) 1
- (c) 3
- (d) Not defined

(iii) **Assertion:** The function $f(x) = |x - 1|$ is continuous at $x = 1$.

Reason: If a function is continuous at a point, it must be differentiable at that point.

- (a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.
- (b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.
- (c) Assertion is true and Reason is false.
- (d) Assertion is false and Reason is true.

(iv) The value of $\int_0^{\pi/2} \log \left(\frac{4+3\sin x}{4+3\cos x} \right) dx$ is:

- (a) 2
- (b) $\frac{3}{4}$
- (c) 0
- (d) 1

(v) The function $f(x) = x^3 - 3x^2 + 4$ is:

- (a) Strictly increasing in $(0,2)$
- (b) Strictly decreasing in $(0,2)$
- (c) Neither increasing nor decreasing in $(0,2)$
- (d) Constant in $(0,2)$

(vi) A spherical balloon is being inflated at the rate of $10 \text{ cm}^3/\text{sec}$. The rate of increase of its surface area when the radius is 5 cm is:

- (a) $2 \text{ cm}^2/\text{sec}$
- (b) $4 \text{ cm}^2/\text{sec}$
- (c) $6 \text{ cm}^2/\text{sec}$
- (d) $8 \text{ cm}^2/\text{sec}$

(vii) Statement 1: Every identity matrix is a scalar matrix.

Statement 2: Every scalar matrix is an identity matrix.

- (a) Statement 1 is true and Statement 2 is false.
- (b) Statement 2 is true and Statement 1 is false.
- (c) Both the statements are true.
- (d) Both the statements are false.

(viii) The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at $x = \frac{1}{2}$ is:

- (a) 2
- (b) -2
- (c) 4
- (d) -4

(ix) If $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$, then the value of $P(A' | B')$ is:

- (a) $\frac{1}{7}$
- (b) $\frac{2}{7}$
- (c) $\frac{5}{7}$
- (d) $\frac{3}{7}$

(x) Find the number of equivalence relations on the set $A = \{1, 2, 3\}$

(xi) The function $f(x) = \sin x - \cos x$ is strictly increasing in the interval:

- (a) $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$
- (b) $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$
- (c) $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$
- (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(xii) A random variable X has the following probability distribution:

X	0	1	2
$P(X)$	0.3	0.5	0.2

Then $E(X^2)$ is:

- (a) 0.9
- (b) 1.3
- (c) 1.7
- (d) 2.1

(xiii) If $R = \{(x, y) : x, y \in \mathbb{Z}, x^2 + y^2 \leq 4\}$ is a relation on the set $\mathbb{Z}$ of integers, write the domain of R .

(xiv) Evaluate: $\int \frac{dx}{x(x^5+1)}$

(xv) A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. What is the probability that the problem will be solved?

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Question 2

Find the point on the curve $y = x^3 - 3x^2 + 2x$ where the tangent is parallel to the line $y = 2x + 5$. [2]

Question 3

Solve for x : $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$ [2]

Question 4

(i) If $y = \log(x + \sqrt{x^2 + 1})$, prove that $(1 + x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0$.

OR

(ii) If $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$. [2]

Question 5

The monthly incomes of two sisters Reema and Seema are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. If each saves ₹5000 per month, find their monthly incomes using matrix method. [2]

Question 6

(i) Differentiate $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$.

OR

(ii) Show that the differential equation $(x-y)\frac{dy}{dx} = x+2y$ is homogeneous and solve it. [2]

Question 7

Using properties of determinants, prove that:

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$
 [4]

Question 8

(i) Evaluate: $\int e^{2x} \sin(3x) dx$

(ii) Evaluate: $\int_{-1}^2 |x^3 - x| dx$ [4]

Question 9

(i) A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 20 m. Find the dimensions of the window to admit the maximum light through the whole opening.

OR

(ii) The curve $y = x^3 - 3x^2 - 9x + 6$ is given.

(a) Find the slope of the curve at $x = 0$. [1]

(b) Find the points on the curve where the tangent is parallel to the x-axis. [2]

(c) Determine the intervals in which the function is strictly increasing. [1] [4]

Question 10

A bag contains 4 red and 4 black balls. Another bag contains 2 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random

(without replacement) from the selected bag.

(i) Find the probability that both balls drawn are red. [2]

(ii) If the balls drawn are of the same colour, what is the probability that they are from the first bag? [2]

Question 11

(i) Prove that: $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx = \frac{\pi^2}{4}$

OR

(ii) Evaluate: $\int \frac{2x}{(x^2+1)(x^2+3)} dx$ [6]

Question 12

(i) Solve the differential equation: $(1 + y^2)dx = (\tan^{-1} y - x)dy$

OR

(ii) Find the particular solution of the differential equation $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$ ($x \neq 0$), given that $y = 0$ when $x = \frac{\pi}{2}$. [6]

Question 13

Consider the function $y = \sin^{-1}(2x\sqrt{1-x^2})$, $x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

(i) Find the range of y if $x = 0, \frac{1}{2}$ and $-\frac{1}{2}$. [1]

(ii) Prove: $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$ [1]

(iii) Evaluate: $\tan^{-1}\left(2\cos\left(\frac{\pi}{3}\right)\left(2\sin^{-1}\frac{1}{2}\right)\right)$ [2]

(iv) If $4\sin^{-1} x + \cos^{-1} x = \pi$, calculate the value of x . [2][6]

Question 14

In a bolt factory, three machines A, B, and C manufacture 25%, 35%, and 40% of the total production respectively. Of their output, 5%, 4%, and 2% are defective bolts respectively. A bolt is drawn at random from the total production and is found to be defective. What is the probability that it was manufactured by machine B? [6]

SECTION B - 15 MARKS

Question 15

In subparts (i) and (ii) choose the correct options and in subparts (iii) and (iv), answer the questions as instructed.

(i) Assertion: The lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar.

Reason: Two lines are coplanar if the shortest distance between them is zero.

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(ii) The angle between the line $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ is:

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(b) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(d) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(iii) Find the vector equation of the plane passing through the points $A(2,1,-1)$, $B(3,4,2)$, and $C(7,0,6)$.

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coordinates of the point where the line $\frac{x-4}{-4} = \frac{y+5}{-3} = \frac{z+1}{1}$ crosses the plane $2x + y + z = 7$.

[4]

Question 16

(i) Find the shortest distance between the lines:

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$$

OR

(ii) Find the equation of the line passing through the point $(-1, 3, -2)$ and perpendicular to the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. [4]

Question 17

If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$, and $\vec{c} = -3\hat{i} + \hat{j} + 2\hat{k}$, then:

(i) Find $\vec{a} \times (\vec{b} \times \vec{c})$. [1]

(ii) Find a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. [2]

(iii) Verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$. [1]

Question 18

(i) Using integration, find the area of the region bounded by the parabola $y^2 = 4x$, the x-axis, and the lines $x = 1$ and $x = 4$.

OR

(ii) Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$. [4]

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SECTION C - 15 MARKS

Question 19

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) For two variables x and y , the regression equations are $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$. The correlation coefficient between x and y is:

- (a) -0.5
- (b) 0.5

- (c) 0.25
(d) -0.25

(ii) The total cost function for a product is given by $C(x) = \frac{x^3}{3} - 4x^2 + 20x + 10$. The level of output at which marginal cost is minimum is:

- (a) 2
(b) 4
(c) 6
(d) 8

(iii) The following table shows the marks obtained by 5 students in Mathematics (x) and Statistics (y).

x	y
2	3
4	5
6	7
8	10
10	12

- (a) Find the regression coefficient b_{yx} .
(b) Estimate the marks in Statistics when marks in Mathematics are 7. [3]

Question 20

(i) The two regression lines are $8x - 10y + 66 = 0$ and $40x - 18y = 214$. The variance of x is 9. Find the mean values of x and y , and the correlation coefficient between x and y .

OR

(ii) For 10 observations on price (x) and supply (y), the following data were obtained:

$$\sum x = 130, \sum y = 220, \sum x^2 = 2288, \sum y^2 = 5506, \sum xy = 3467.$$

Find the line of regression of y on x .

[4]

Question 21

(i) The total revenue function for a commodity is $R = 40x - 4x^2$ and the total cost function is $C = 2x^2 + 4x + 10$, where x is the level of output.

(a) Find the marginal revenue.

(b) Find the average cost.

(c) Determine the level of output at which profit is maximized.

(d) Calculate the maximum profit.

OR

(ii) If the total cost function is given by $C(x) = x^3 - 3x^2 + 7x + 10$, find:

(a) The Average Cost (AC) function.

(b) The Marginal Cost (MC) function.

(c) The level of output at which AC is minimum.

(d) The minimum Average Cost.

[4]

Question 22

A manufacturer produces two types of toys, A and B. Toy A requires 2 hours of cutting and 1 hour of assembling. Toy B requires 1 hour of cutting and 2 hours of assembling. There are 100 hours available for cutting and 120 hours for assembling. The profit on each toy A is ₹50 and on each toy B is ₹60. Formulate this as a Linear Programming Problem to maximize the profit and solve it graphically.

[4]